

上海交通大学国际本科生招生考试数学科考试试卷

(样卷)

SJTU International Undergraduate Entrance Examination

(Mathematics sample exam papers)

第一部分: 下列问题有且仅有一个正确答案 (每题 3 分, 共 42 分)

Section 1: The following problems have one and only one correct answer.

(3 points for each, 42 points total)

1. 设集合  $A = \{1,2,4\}, B = \{2,4,5\}$ , 则  $A \cap B = ( )$ .

If set  $A = \{1,2,4\}, B = \{2,4,5\}$ , then  $A \cap B = ( )$ .

(A)  $\{2,4\}$  (B)  $\{1,2,4,5\}$  (C)  $\{1\}$  (D)  $\{1,5\}$

2. 函数  $y = (x-1)^{\frac{1}{2}} + (4-x)^{-\frac{3}{2}}$  的定义域为  $( )$ .

The domain of the function  $y = (x-1)^{\frac{1}{2}} + (4-x)^{-\frac{3}{2}}$  is  $( )$ .

(A)  $[1,4)$  (B)  $(-\infty, 1)$  (C)  $[4, +\infty)$  (D)  $(1, 4]$

3. 下列公式中, 正确的是  $( )$ .

In the following formulae, the one that must be correct is  $( )$ .

(A)  $\cos 2x = 2\cos^2 x - 1$  (B)  $\cos 2x = 2\sin^2 x - 1$

(C)  $\sin 2x = 2\cos^2 x - 1$  (D)  $\sin 2x = 2\sin^2 x - 1$

4. 设  $m$  是实常数. 若直线  $l_1: 2x + my + 1 = 0$  与直线  $l_2: y = 3x - 1$  平行, 则  $m = ( )$ .

Let  $m$  be a real number. If line  $l_1: 2x + my + 1 = 0$  is parallel to line  $l_2: y = 3x - 1$ , then  $m = ( )$ .

(A)  $-\frac{2}{3}$  (B)  $\frac{2}{3}$  (C) 6 (D) -6

5. 设平面上的动点  $P$  到定点  $F(2,0)$  的距离等于  $P$  到直线  $x + 2 = 0$  的距离, 则点  $P$  的轨迹方程为  $( )$ .

If the distance from moving point  $P$  to point  $F(2,0)$  equals to the distance from  $P$  to the straight line  $x + 2 = 0$ , then the trajectory equation of  $P$  is  $( )$ .

(A)  $y^2 = 8x$  (B)  $y^2 = -8x$  (C)  $x^2 = 8y$  (D)  $x^2 = -8y$

6. 下列函数中, 在其定义域上是单调递减的函数是  $( )$ .

In the following functions, the one that is decreasing in its domain is  $( )$ .

(A)  $y = 2^{-x}$  (B)  $y = \cot x$  (C)  $y = \frac{1}{x^2+1}$  (D)  $y = x$

7. 下列选项中, 正确的是 ( ).

Among the following options, the correct one is ( ).

(A)  $y = x^3 + \frac{1}{x}$  和  $y = \log_2(x + \sqrt{x^2 + 1})$  均是奇函数

$y = x^3 + \frac{1}{x}$  and  $y = \log_2(x + \sqrt{x^2 + 1})$  are both odd functions

(B)  $y = x^3 + \frac{1}{x}$  是奇函数, 但  $y = \log_2(x + \sqrt{x^2 + 1})$  不是奇函数

$y = x^3 + \frac{1}{x}$  is an odd function but  $y = \log_2(x + \sqrt{x^2 + 1})$  is not an odd function

(C)  $y = \log_2(x + \sqrt{x^2 + 1})$  是奇函数, 但  $y = x^3 + \frac{1}{x}$  不是奇函数

$y = \log_2(x + \sqrt{x^2 + 1})$  is an odd function but  $y = x^3 + \frac{1}{x}$  is not an odd function

(D)  $y = x^3 + \frac{1}{x}$  和  $y = \log_2(x + \sqrt{x^2 + 1})$  均不是奇函数

neither  $y = x^3 + \frac{1}{x}$  nor  $y = \log_2(x + \sqrt{x^2 + 1})$  is an odd function

8. 已知  $\{a_n\}$  是等差数列, 且  $a_2 = 12, a_8 = 18$ , 则  $a_5 = ( )$ .

Given that  $\{a_n\}$  is an arithmetic sequence, and  $a_2 = 12, a_8 = 18$ , then  $a_5 = ( )$ .

(A) 15 (B)  $6\sqrt{6}$  (C) 30 (D) 216

9. 若函数  $f(x)$  的反函数  $f^{-1}(x) = x^2 (x > 0)$ , 则  $f(4) = ( )$ .

If the inverse function of  $f(x)$  is  $f^{-1}(x) = x^2 (x > 0)$ , then  $f(4) = ( )$ .

(A) 2 (B) -2 (C) 16 (D) -16

10.  $\arctan\left(\tan\frac{5\pi}{6}\right) = ( )$ .

(A)  $-\frac{\pi}{6}$  (B)  $\frac{\pi}{6}$  (C)  $-\frac{5\pi}{6}$  (D)  $\frac{5\pi}{6}$

11. 已知椭圆  $\Gamma: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  的一个焦点是  $(-2\sqrt{3}, 0)$ . 若  $a = 2b$ , 则  $b = ( )$ .

Let  $(-2\sqrt{3}, 0)$  be one focus of the ellipse  $\Gamma: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . If  $a = 2b$ , then  $b = ( )$ .

(A) 2 (B)  $\frac{\sqrt{60}}{5}$  (C) 4 (D)  $\frac{12}{5}$

12. 设 $a, b$  均为正实数, 则“ $a + b \leq 2$ ” 是“ $a^2 + b^2 \leq 2$ ” 的 ( ) .

Let  $a, b$  be positive real numbers, then statement “ $a + b \leq 2$ ” is a ( ) for statement “ $a^2 + b^2 \leq 2$ ”.

(A) 必要但非充分条件

necessary but not sufficient condition

(B) 充分但非必要条件

sufficient but not necessary condition

(C) 既非充分又非必要条件

neither sufficient nor necessary condition

(D) 充分且必要条件

sufficient and necessary condition

13. 函数 $\sin(2x + 3)$  的导数是 ( ) .

The derivative of function  $\sin(2x + 3)$  is ( ) .

(A)  $2 \cos(2x + 3)$  (B)  $\cos(2x + 3)$  (C)  $-2 \cos(2x + 3)$  (D)  $-\cos(2x + 3)$

14. 曲线  $y = x^3 + 3x + 1$  在点  $P(0,1)$  处的切线方程是 ( ) .

The tangential equation of curve  $y = x^3 + 3x + 1$  at point  $P(0,1)$  is ( ) .

(A)  $3x - y + 1 = 0$  (B)  $x - 3y + 3 = 0$  (C)  $3x + y - 1 = 0$  (D)  $x + 3y - 3 = 0$

**第二部分: 下列问题有且仅有一个正确答案 (每题 4 分, 共 48 分)**

**Section 2: The following problems have one and only one correct answer.**

**(4 points for each, 48 points total)**

15. 不等式  $\frac{1}{x-2} < \frac{1}{x}$  的解集为 ( ) .

The solution set of the inequality  $\frac{1}{x-2} < \frac{1}{x}$  is ( ) .

(A) (0,2) (B) (1,3) (C)  $(-\infty, -1)$  (D)  $(2, +\infty)$  (E)  $(-1,0)$

16. 已知等比数列 $\{a_n\}$ 的首项  $a_1 = 1$ , 公比  $q = 2$ , 则 $\{a_n\}$ 的前 8 项的和  $S_8 =$  ( ) .

If the first term  $a_1$  of the geometric sequence  $\{a_n\}$  is 1, and the common quotient  $q = 2$ , then the sum  $S_8$  of the first 8 terms of  $\{a_n\}$  is ( ) .

(A) 255 (B) 127 (C) 63 (D) 511 (E) 512

17. 已知实数  $a, b$  满足  $2^{2a-b} = 4^{a+b} = 3$ , 则  $a =$  ( ) .

If real numbers  $a, b$  satisfy  $2^{2a-b} = 4^{a+b} = 3$ , then  $a = ( )$ .

- (A)  $\log_4 3$  (B)  $\log_2 3$  (C)  $\log_3 2$  (D)  $\log_3 4$  (E)  $\log_4 2$

18. 已知  $a, b, 1, 2$  的中位数是 3, 平均数是 4, 则  $ab = ( )$ .

Let the median of  $a, b, 1, 2$  be 3, the average of  $a, b, 1, 2$  be 4, then  $ab = ( )$ .

- (A) 36 (B) 22 (C) 30 (D) 40 (E) 42

19. 已知点  $P$  在曲线  $C: 2x^2 - 4x + 2y^2 - 12y = 5$  上, 点  $Q$  在直线  $x + y + 3 = 0$  上, 则点  $P$  和  $Q$  之间点距离  $|PQ|$  的最小值为  $( )$ .

If point  $P$  is on the curve  $C: 2x^2 - 4x + 2y^2 - 12y = 5$ , and point  $Q$  is on the line  $x + y + 3 = 0$ , then the minimum distance  $|PQ|$  between points  $P$  and  $Q$  is  $( )$ .

- (A)  $\sqrt{2}$  (B)  $\frac{\sqrt{2}}{2}$  (C) 1 (D) 0 (E) 2

20. 设复数  $z = \sqrt{3} + i$ , 其中  $i$  是虚数单位, 则  $z^5 = ( )$ .

Let complex number  $z$  be defined as  $z = \sqrt{3} + i$ , where  $i$  is the unit of imaginary numbers, then  $z^5 = ( )$ .

- (A)  $-16\sqrt{3} + 16i$  (B)  $16\sqrt{3} + 16i$  (C)  $-16\sqrt{3} - 16i$   
(D)  $16\sqrt{3} - 16i$  (E)  $16\sqrt{3} + 16\sqrt{3}i$

21. 在所有两位数中, 个位数和十位数之和是偶数的数有  $( )$ .

In all two-digit numbers, the number of those which the sum of its ones digit and its tens digit is an even number is  $( )$ .

- (A) 45 (B) 25 (C) 40 (D) 20 (E) 50

22. 若实数  $x, y$  满足  $\sin x \cos y = \frac{4}{5}$ ,  $\sin y \cos x = \frac{1}{5}$ , 则  $\cos 2x = ( )$ .

If real numbers  $x, y$  satisfy  $\sin x \cos y = \frac{4}{5}$ ,  $\sin y \cos x = \frac{1}{5}$ , then  $\cos 2x = ( )$ .

- (A)  $-\frac{3}{5}$  (B)  $\frac{3}{5}$  (C)  $\frac{4}{5}$  (D)  $-\frac{4}{5}$  (E)  $\frac{2}{5}$

23. 三位同学参加跳过、跳远和铅球项目的比赛. 若每人都选择两个项目, 则有且仅有两人选择的项目相同的概率是  $( )$ .

Three students participated in high jump, long jump and shot put competitions. If everyone chooses two of these three items, then the probability that there are exactly two people choosing same items is  $( )$ .

- (A)  $\frac{2}{3}$  (B)  $\frac{1}{3}$  (C)  $\frac{2}{9}$  (D)  $\frac{1}{9}$  (E)  $\frac{1}{6}$

24. 设双曲线  $C: x^2 - \frac{y^2}{3} = 1$  的左右焦点分别为  $F_1$  和  $F_2$ . 若点  $P$  在  $C$  上, 且

$$\frac{\sin \angle PF_2F_1}{\sin \angle PF_1F_2} = 2, \text{ 则 } \cos \angle F_1PF_2 = ( \quad ).$$

Let the foci of the hyperbola  $C: x^2 - \frac{y^2}{3} = 1$  be  $F_1$  and  $F_2$ , respectively. If point  $P$  is on

$C$ , such that  $\frac{\sin \angle PF_2F_1}{\sin \angle PF_1F_2} = 2$ , then  $\cos \angle F_1PF_2 = ( \quad )$ .

- (A)  $\frac{1}{4}$       (B)  $-\frac{1}{2}$       (C)  $\frac{1}{6}$       (D)  $-\frac{1}{5}$       (E)  $\frac{1}{3}$

25. 设  $P-ABC$  是棱长为 6 的正四面体, 点  $D, E, F$  分别是三角形  $\Delta PAB, \Delta PBC, \Delta PAC$  的重心, 则三棱锥  $P-DEF$  的体积为 ( ).

Let  $P-ABC$  be a tetrahedron whose edges have a length of 6. If points  $D, E, F$  are the barycenter of triangles  $\Delta PAB, \Delta PBC, \Delta PAC$  respectively, then the volume of the trigonal pyramid  $P-DEF$  is ( ).

- (A)  $\frac{4\sqrt{2}}{3}$       (B)  $\frac{\sqrt{2}}{3}$       (C)  $\frac{3\sqrt{2}}{2}$       (D)  $2\sqrt{2}$       (E)  $4\sqrt{2}$

26. 设数列  $\{a_n\}, \{b_n\}$ , 和  $\{c_n\}$  满足: 对任意正整数  $n$ ,  $a_{n+1} = (-1)^n(a_n^2 + 1)$ ,  $b_n = a_n a_{n+1}$ ,  $c_n = \cos a_n$ . 下列论断中正确的的是 ( ).

Let sequences  $\{a_n\}, \{b_n\}$ , and  $\{c_n\}$  satisfy: for an arbitrary integer  $n$ ,  $a_{n+1} = (-1)^n(a_n^2 + 1)$ ,  $b_n = a_n a_{n+1}$ ,  $c_n = \cos a_n$ , then the one that must be correct in the following statements is ( ).

- (A)  $\{b_n\}$  是单调递减数列  
 $\{b_n\}$  is a decreasing sequence
- (B)  $\{b_n\}$  是单调递增数列  
 $\{b_n\}$  is an increasing sequence
- (C)  $\{c_n\}$  是单调递增数列  
 $\{c_n\}$  is an increasing sequence
- (D)  $\{a_n\}$  是单调递增数列  
 $\{a_n\}$  is an increasing sequence
- (E)  $\{c_n\}$  是单调递减数列  
 $\{c_n\}$  is a decreasing sequence

第三部分：下列问题有且仅有一个正确答案（每题 5 分，共 10 分）

Section 3: The following problems have one and only one correct answer.

(5 points for each, 10 points total)

27. 设  $A, B, C$  是三角形  $\triangle ABC$  的三个顶点，且对任意的实数  $\lambda$  恒有  $|\overrightarrow{BA} - \lambda \overrightarrow{BC}| \geq 2|\overrightarrow{BC}|$ . 若  $|\overrightarrow{BC}| = 1$ , 则三角形  $\triangle ABC$  周长的最小值为 ( ).

Let points  $A, B, C$  be the three vertices of triangle  $\triangle ABC$ , such that for an arbitrary real number  $\lambda$ , the following always holds true:  $|\overrightarrow{BA} - \lambda \overrightarrow{BC}| \geq 2|\overrightarrow{BC}|$ . If  $|\overrightarrow{BC}| = 1$ , then the minimum value of the perimeter of the triangle  $\triangle ABC$  is ( ).

- (A)  $1 + \sqrt{17}$  (B)  $1 + \sqrt{12}$  (C)  $1 + \sqrt{7}$  (D)  $1 + \sqrt{22}$  (E)  $1 + \sqrt{27}$

28. 已知实数  $x, y, z$  满足  $x^2 + y^2 + z^2 = 1$ , 则  $xy + 2yz$  的最大值为 ( ).

If  $x, y, z$  are all real numbers, and  $x^2 + y^2 + z^2 = 1$ , then the maximum value of  $xy + 2yz$  is ( ).

- (A)  $\frac{\sqrt{5}}{2}$  (B)  $\frac{\sqrt{2}}{2}$  (C)  $\frac{\sqrt{3}}{2}$  (D) 2 (E)  $\frac{1}{2}$

参考答案：

1~28：A