Syllabus for SJTU International Undergraduate Entrance Examination

(Mathematics)

(NOV. 2021)

I. Purpose of the test

Mathematics plays an irreplaceable role in forming rational thinking and scientific spirit and promoting the development of individual intelligence. The International Undergraduates Entrance Examination (Mathematics) (IUEE-Mathematics) is to comprehensively test whether the candidates have mastered the mathematical knowledge, skills, thoughts, and methods which are necessary for further study, to understand whether the candidates can observe the world with mathematical vision, to think the world with mathematical mind and to express the world with mathematical language.

II. Skills assessed in the test

The test focuses on the candidates' basic knowledge, skills, and thoughts of mathematics, and makes a preliminary evaluation on the candidates' ability to discover and construct problems, to analyze and solve problems from the perspective of mathematics.

1. Basic Mathematical Knowledge and Skills:

a) To comprehend or grasp the basic knowledge in elementary mathematics involving numbers and operations, equations and algebra, functions and analysis, data collation and probability statistics, as well as graphics and geometry.

b) To understand the basic mathematical ideas of set, mapping, function, algorithm, mathematical modeling, limit, probability, statistics, normalization, shape-number combination, classification, decomposition, and composition, and so on; to master the basic skills such as comparison, analysis, analogy, induction, coordinate method, parameter method, logical partition, equivalent transformation, etc.

c) To be able to calculate, draw and reason according to certain rules and steps; to master the basic skills of mathematical reading and expression as well as the conversion between text language, graphic language, and symbol language.

2. *Mathematical Abstraction Ability:*

   To be able to formulate the mathematical research object by abstracting the quantity relation and the spatial form, mainly including abstracting the mathematical concept and the relation between concepts from the quantity and quantity relation as well as graph and graph relation, abstracting the general rule and structure from the specific background of things, and characterizing it by mathematical language.

3. *Logical Reasoning Ability:*

   To be able to judge causality correctly, and based on rules, to deduce other propositions from some facts and propositions, and to describe the reasoning process correctly and concisely, which mainly includes two types: one is the reasoning from the special to the general, in the form of induction and analogy; the other one is the reasoning from the general to the special, in the form of deduction.

4. *Intuitive Imagination Ability:*

   To be able to perceive the shape and change of things with the aid of geometric intuition and spatial imagination, to understand and solve mathematical problems using spatial forms, especially graphics, which mainly includes: to learn the positional relation, the morphological changes, and law of motion in objects with the help of spatial forms; to describe and analyze mathematical problems using graphics; to establish the relation between shape and number, to construct the straightforward model of mathematical problems, and to explore ways to solve problems.

5. *Mathematical Operation Ability:*

   To be able to solve mathematical problems according to operation rules based on a clear operand, which mainly include understanding the operation object, mastering operation rules, exploring operation ideas, selecting operation methods, designing operation process, and obtaining operation results, etc.

6. *Mathematical Abilities of Application and Exploration:*

   a) To be able to utilize basic mathematical knowledge, skills, ideas, and methods as well as appropriate problem-solving strategies to solve mathematical problems.

   b) To be able to solve practical problems by mathematical modeling and to explain

the real significance therein.

c) To be able to use relevant mathematical ideas and methods to explore problems and correctly describe processes and results.

III. Structure of the test

The duration of this test is 60 minutes, and the total score is 100. The content and structure of the test are presented in Table below.

The content and structure of the IUEE-Mathematics

Knowledge and Skills	**Response format**	**Number of items**	**Weight (%)**	**Time（minutes）**
Prerequisites	Multiple-choice questions	4	14	**60**
Function	Multiple-choice questions	13	44	
Geometry and algebra	Multiple-choice questions	8	30	
Probability and Statistics	Multiple-choice questions	8	12	
Total			**100**	

IV. Format of the test

The IUEE-Mathematics is a computer-based test. All question types are presented on the computer, and students are required to complete all the tasks on the computer. The prompts and questions are presented in the form of audios, videos, texts, and graphics

V. Scoring and score reporting

Multiple choice items are automatically scored by the computer. The IUEE-Mathematics reports a total score of 0 to 100.

Appendix 1. Knowledge and Skills

I. Prerequisites

1. Set: concept and representation of sets; basic relations of sets; basic operations of sets like intersection, union, and complement, etc.
2. Common logical terms: necessary condition; sufficient condition; necessary and sufficient condition; universal quantifiers and existential quantifiers; negation of universal quantifier propositions; negation of existential quantifier propositions.
3. Equalities and inequalities: properties of equalities and inequalities; basic inequalities.
4. Solving equations and inequalities: solving quadratic equations and inequalities of one variable; solving equations and inequalities with absolute values; solving fractional inequalities, etc.

II. Function

1. Concept and properties of functions: concept of functions; arithmetic operations of functions; composition of functions; monotonicity of functions; maximum and minimum values of functions; parity of functions; periodicity of functions; geometric interpretation of function properties.
2. Inverse functions: concept of inverse functions; inverse functions of elementary functions; the connections and differences between a function and its inverse function; the relationship between the graphs of functions and their inverse functions.
3. Power functions, exponential functions, logarithmic functions: the concept, operations and properties of power functions; the concept, operations and properties of exponential functions; the concept, operations and properties of logarithmic function; Change-of-Base formula of logarithms.
4. Trigonometric functions: concepts and properties of trigonometric functions; basic trigonometric identities; double-angle identities; half-angle identities; angle sum and difference formula of two angles; sum-to-product and product-to-sum identities; tangent half-angle formulas; trigonometric transformations using identities.
5. Inverse trigonometric functions: concepts and properties of inverse trigonometric functions; the relationship between inverse trigonometric functions and trigonometric functions; calculation of the values of inverse trigonometric functions.

6. Applications of functions: choosing the appropriate function type to describe the patterns of real-life problems.
7. Sequences: the concept of a sequence; mathematical induction.
8. Arithmetic sequences: the concept of an arithmetic sequence; the general term formula; the formula for the sum of the first n terms; the relationship between arithmetic sequences and one-variable linear functions.
9. Geometric sequences: the concept of a geometric sequence; the general term formula; the formula for the sum of the first n terms; the relationship between geometric sequences and exponential functions.
10. Properties of sequences: bounded sequences; periodic sequences; monotonicity of sequences and its applications.
11. Simple recursive sequences: determining the general term formula and the formula for the sum of the first n terms of a sequence by a simple recurrence relation.
12. Derivatives of one-variable functions and their applications: the concept of derivative and its geometric interpretation; arithmetic operations of derivatives; derivatives of composite functions; the relationship between monotonicity of a function and its derivative; the necessary and sufficient conditions for a function to obtain extreme values at certain points; the maximum and minimum values of a function on a given closed interval.

III. Geometry and algebra

1. Vectors and their applications: the concept of vectors; vector operations; the Fundamental Theorem of Vectors and coordinate representations of vectors; orthogonal decompositions of vectors; vector projections and the projection vector; sine and cosine rules.
2. Complex numbers: complex numbers and related concepts; operations of complex numbers; trigonometric representations of complex numbers.
3. Surface areas and volumes of solids: formulas for the surface areas and volumes of spheres, prisms, cylinders, pyramids, cones, and frustums, etc.
4. Points, lines, and planes, and the distances and angles between each other: the positional relationships of points, straight lines and planes; property and decidability theorems of points, straight lines and planes; The distances and angles between two lines, a line and a plane, and two planes.
5. Equations of straight lines, the positional relationship between straight lines: calculation of the slope of a straight line; determining whether two lines are parallel or perpendicular; several forms of equations of a line (point-slope form, two-point form, general form, etc.); the coordinates of the intersection point of two straight lines; the formula of distance between two points; the

formula of distance from a point to a straight line.

6. Equations of circles, positional relationship between a line and a circle, and between two circles: the standard and general equations of a circle; determining the positional relationship between a straight line and a circle, and between two circles.
7. Definitions, the standard equations and simple geometric properties of conic curves: the definitions, the standard equations, simple geometric properties, points of intersection with straight lines, and simple applications of three types of conic curve like ellipses, parabolas and hyperbolas, respectively.

IV. Probability and Statistics

1. Counting principles: the addition and multiplication principles; permutations and combinations; permutation and combination formulas; the Binomial Theorem.

2. Probability: independence of random events; the conditional probability of random events; probabilities of random events in classical probability; the law of total probability; the Bayes formula; discrete random variables and their distribution sequences; the binomial distribution, the hypergeometric distribution, the normal distribution and their numeric characteristics; solving simple practical problems.

3. Statistics: basic ways to obtain data and corresponding concepts; sampling; statistical charts; estimating population statistics with samples statistics (mean, median, mode, standard deviation, variance, range); statistical correlation and the correlation coefficient of paired data sets; the simple linear regression model; the least squares method; the independence test of a 2×2 contingency table and its applications.